Robust and Decoupled Position and Stiffness Control for Electrically-Driven Articulated Soft Robots

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***Abstract*—The control of articulated soft robots, i.e. robots with flexible joints and rigid links, presents a challenge due to their intrinsic elastic elements and nonlinear force-deflection dependency. This letter first proposes a discrete-time delayed unknown input-state observer based on a nominal robot model that reconstructs the total torque disturbance vector, resulting from the imperfect knowledge of the elastic torque characteristic, external torques, and other model uncertainties. Then, it introduces a robust controller, that actively compensates for the estimated uncertainty and allows bounded stability for the tracking of independent link position and joint stiffness reference signals. The convergence of the disturbance estimator and the overall system's stability in closed loop is proven analytically, while the effectiveness of the proposed control design is first evaluated in simulations with respect to large uncertainty conditions, and then demonstrated through experiments on a real multi-degree-of-freedom articulated soft robot.**

***Index Terms*—Robust/adaptive control, flexible robotics, compliance and impedance control.**

I. Introduction

ENDOWED with intrinsic flexibility, Articulated Soft Robots (ASR) can reach competitive skills, such as adaptation to unstructured environment [1], effective energy storage and release [2], and stable interaction with static environment [3], that are typical of biomechanical systems. To fully match the skills of vertebrates, a class of articulated soft robots with Variable Stiffness Actuation (VSA) technology has been developed, capable of modifying robot joint stiffness with time [4]. The potential capacity to regulate simultaneously, and in a decoupled manner, joint position and stiffness [5]–[7], empowers them with even more dynamic manipulation capabilities, allowing them to perform tasks that involve for example throwing and catching [8] of objects, that were not attainable by rigid robots just a few decades ago.

Being biologically inspired and of a less complex mechanical design compared to other VSAs, agonistic-antagonistic VSAs are often applied technology in ASR systems [9]. However, the gained advantage of a simpler mechanical design is traded-off with the need for a more elaborated controller that can allow decoupled position and stiffness regulation. Moreover, the agonistic-antagonistic actuators introduce nonlinearities into the model since a nonlinear force-deformation dependency is needed to achieve a time-varying stiffness [10], while the existence of elastic tendons introduces hysteresis and additional nonlinearities since tendons change their characteristic over time due to wear-off and working temperature, or even break [11].

To tackle the challenge of controlling an ASR, the first line of research consists of model-based approaches, which leverage on precise knowledge of either the full robot dynamics or the actuator's model. Relevant examples of the first setting are the ones obtained by feedback linearization [5], gain-scheduling control [12], and backstepping control [13], while adaptive control [7] and control based on damping injection [14] have been used in the latter one. A second, also very promising, line of research is model-free and involves techniques that aim at iteratively learning the position or torque control of an ASR, by using the minimum knowledge about the system while preserving its compliance through a feedforward control component [6], [15], yet without guaranteeing stability. Moreover, compared to the state feedback control in [16] this letter avoids using torque sensors.

The present work capitalizes on the advantages of model-based formal approaches to provide stability guarantees, and, in that regard, it proposes a novel disturbance-based robust control solution for articulated soft robots with antagonistic VSA, that ensures the performance even under lack of system knowledge or presence of the external disturbance. More precisely, our approach assumes arbitrary values of the parameters that construct the robot's inertia matrix, chosen under the constraint that the inertia matrix is non-singular. The proposed solution builds upon the theory of delayed Unknown Input Observers (UIO), which allows estimating and compensating the model's uncertainties after only a few samples [17], compared to the dozen iterations needed for learning-based approaches. Compared to other existing solutions such as Extended State Observers (ESO), UIOs do not lean on assumptions on the disturbance dynamics and its boundedness, require no parameter tuning, have exact convergence guarantees, and outperform them [18] despite their simplicity. Moreover, the controllers' robustness to the robot

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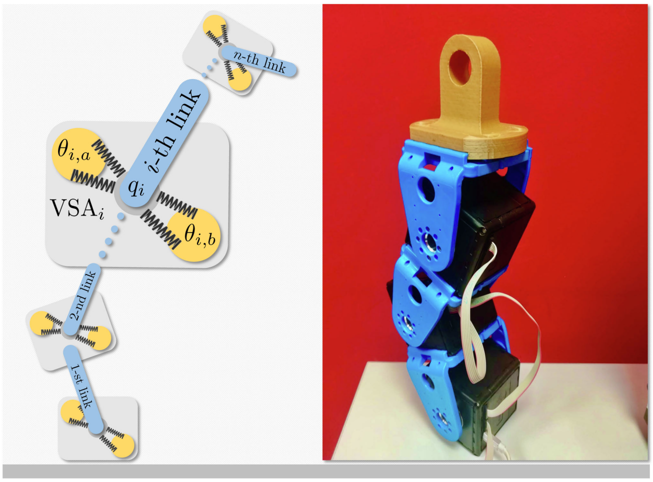

Fig. 1. Depiction of an *articulated soft robot* with n rigid links and n flexible joints driven by electromechanical VSA devices (left) and picture of the 3-degree-of-freedom hardware setup used to validate the proposed solution (right).

and actuator model uncertainties facilitates its application and avoids the necessity for extensive a-priori model identification.

The contributions of the letter are: 1) the formalization of a model for VSA-driven articulated soft robots with factorized actuator dynamics, allowing a convenient decomposition into arbitrarily simple yet invertible nominal and uncertain dynamics; 2) a formulation of a delayed UIO for VSA-driven robots that simultaneously estimates the input disturbance and system states, and, consequently, avoids the necessity to collect velocity and acceleration measures at the only cost of a few sample delay; 3) the design of a robust composite disturbance-observer-based controller, which uses the information on state and input estimates to provide perfect asymptotic tracking of position and stiffness desired trajectories; 4) simulation and experimental validations on a multi-degree-of-freedom VSA-driven articulated soft robot that prove the effectiveness of the proposed control design in successfully tracking the desired position and stiffness references despite the existing nonlinearities and uncertainties.

II. Background and Problem Statement

The structure of an articulated soft robot with n links and n flexible joints, driven by electromechanical VSA devices, is illustrated in Fig. 1. The actuation of the i-th robot joint is obtained as the result of mechanical deflections of elastic elements within each VSA, which are generated by internal pairs of electric motors. Such pairs are arranged in so-called agonistic-antagonistic configuration so as to enable simultaneous setting of the link position and joint stiffness. Indicating with q_i the i-th link position and with $\theta_{i,a}$ and $\theta_{i,b}$ the internal motor positions of the i-th VSA device, the i-th pair of deflections are

$$\phi_{i,a} = q_i - \theta_{i,a}\,, \quad \phi_{i,b} = q_i - \theta_{i,b}\,, \tag{1}$$

and produce an agonistic elastic torque $\tau^e_{i,a}(\phi_{i,a})$ and an antagonistic elastic torque $\tau^e_{i,b}(\phi_{i,b})$, that are simultaneously applied at the i-th link. The total elastic torque applied at the i-th link is then

$$\tau^e_i = \tau^e_{i,a}(\phi_{i,a}) + \tau^e_{i,b}(\phi_{i,b})\,. \tag{2}$$

Defining the robot configuration vector $q = (q_1, \dots, q_n)^T$, the motor position vectors and elastic torque vectors, $\theta_j = (\theta_{1,j}, \dots, \theta_{n,j})^T$ and $\tau^e_j = (\tau^e_{1,j}, \dots, \tau^e_{n,j})^T$, for $j = a, b$, respectively, and the total elastic torque vector $\tau^e = \tau^e_a + \tau^e_b$, the robot's dynamic model, including the link position dynamics and those of the motors within the VSA devices, is given by [10]:

$$\begin{aligned} M(q)\,\ddot q + h(q,\dot q) + \tau^e(\phi_a,\phi_b) &= \tau_{\text{ext}}\,,\\ J_a\,\ddot\theta_a + \Delta_a\,\dot\theta_a - \tau^e_a(\phi_a) &= \tau_a\,,\\ J_b\,\ddot\theta_b + \Delta_b\,\dot\theta_b - \tau^e_b(\phi_b) &= \tau_b\,. \end{aligned} \tag{3}$$

where $M \in \mathbb{R}^{n\times n}$ is the robot's inertia matrix, $h(q,\dot q) \in \mathbb{R}^n$ is a vector field collecting the centrifugal, Coriolis, and gravity terms, $\tau_{\text{ext}} \in \mathbb{R}^n$ is an externally applied torque vector, J_a and J_b are the motors' inertia, Δ_a and Δ_b are damping coefficients, and τ_a and τ_b are the electrically-induced motor torques. Accordingly, the i-th joint stiffness is given by definition

$$\sigma_i = \tfrac{\partial}{\partial\phi_{i,a}}\,\tau^e_{i,a}(\phi_{i,a}) + \tfrac{\partial}{\partial\phi_{i,b}}\,\tau^e_{i,b}(\phi_{i,b})\,, \tag{4}$$

and the joint stiffness matrix is $\sigma = \operatorname{diag}(\sigma_1, \dots, \sigma_n)$. Within this setting, the present letter addresses the following problem:

Problem 1: Given an ASR with dynamics as in (3) and joint stiffness as in (4), design a robust observer-based controller ensuring simultaneous and asymptotic tracking of desired link position and joint stiffness signals, $q_d(t)$ and $\sigma_d(t)$.

Before moving on to the letter contributions, the following definitions and properties are recalled. First, a square matrix A is *Schur* if all its eigenvalues are within the unit circle, i.e. $|\lambda_i(A)| < 1$ for all i. The notation $0_{\ell\times h}$ represents a null matrix belonging to $\mathbb{R}^{\ell\times h}$, while 0_ℓ is used as a short-hand when $h = \ell$. Also, I_ℓ indicates the identity matrix of order ℓ. Moreover, given a linear discrete-time model of the form

$$X_{k+1} = A\,X_k + B\,U_k\,, \quad Y_k = C\,X_k + D\,U_k\,, \tag{5}$$

where k is a discrete time step, $X_k \in \mathbb{R}^n$ is a state vector, $U_k \in \mathbb{R}^m$ an unknown input vector, and $Y_k \in \mathbb{R}^r$ an output vector, the state matrix is $A \in \mathbb{R}^{n\times n}$, input matrix is $B \in \mathbb{R}^{n\times m}$, output matrix is $C \in \mathbb{R}^{r\times n}$ and feed-through matrix is $D \in \mathbb{R}^{r\times m}$ the L-step invertibility and observability matrices are obtained by the recursive definitions

$$\mathbb{H}_L = \begin{pmatrix} D & 0_{r\times m}\\ \mathcal{O}_{L-1}B & \mathbb{H}_{L-1}\end{pmatrix} \text{ and } \mathcal{O}_L = \begin{pmatrix} C\\ \mathcal{O}_{L-1}\,A\end{pmatrix},$$

for $L \geq 1$ and $\mathbb{H}_0 = D$ and $\mathcal{O}_0 = C$. Given the sequences of the matrices, $\mathbb{H}_L$ and $\mathcal{O}_L$, for $L = 0, 1, \dots, n$, a dynamic model as in (5) is said to be *strongly observable* if, and only if, for some L, it holds

$$\operatorname{rank}([\mathcal{O}_L, \mathbb{H}_L]) - \operatorname{rank}(\mathbb{H}_L) = n\,, \tag{6}$$

and it is said to be *invertible* if, and only if, for some L, it holds

$$\operatorname{rank}(\mathbb{H}_L) - \operatorname{rank}(\mathbb{H}_{L-1}) = m\,. \tag{7}$$

III. Compound Configuration Dynamics of ASR - Factorization and Decomposition

The purpose of this section is twofold: first, to describe a convenient and general way to factorize the actuation-related terms, in the position and stiffness dynamics of an articulated soft robot, with respect to a generic basis of functions; then, to introduce a general decomposition of the obtained model as an arbitrarily simple yet invertible nominal dynamics, affected by uncertain input signals.

A. Actuation Model Factorization

Virtually all VSA devices are provided with an integrated and fast control loop allowing a practically instantaneous regulation of the actuation motors. Based on this, the agonistic and antagonistic motor positions, indicated in our model by the vectors θ_a and θ_b, can be viewed as input variables for the robot dynamics. As a result, the robot model in (3) can be restricted to the link equation only:

$$M(q)\,\ddot{q} + h(q,\dot{q}) + \tau^e(\phi_a,\phi_b) = \tau_{\text{ext}}\,, \tag{8}$$

where $\phi_j = (\phi_{1,j},\ldots,\phi_{n,j})^T$, for $j = a,b$, with each $\phi_{i,j} = q_i - \theta_{i,j}$ is the j-th deflection at the i-th joint.

Assuming that all VSA devices in the robot are homogeneous, i.e. they are made of similar copies of the same agonistic/antagonistic actuation mechanism, the entries of the elastic torque vectors τ^e_a and τ^e_b can be expressed as combinations of suitable basis functions, $\{y_1,\ldots,y_p\}$, of the (i,j)-th pair of deflection variables $\phi_{i,j}$ (cf. e.g. the basis choice in [19]), i.e.

$$\tau^e_{i,a} = \sum_{k=1}^{p} \alpha_{i,k}\, y_k(\phi_{i,a})\,,\ \tau^e_{i,b} = \sum_{k=1}^{p} \beta_{i,k}\, y_k(\phi_{i,b}),$$

for $i = 1,\ldots,n$, with $\alpha_{i,k}$ and $\beta_{i,k}$ being the coefficients of the combinations. Note that since the functions y_k belong to a basis, they can be assumed to be linearly independent. From (2) and considering the separability of the elastic torque factors, the total elastic torque vector τ^e can be factorized as in the following:

$$\tau^e = \begin{pmatrix} \tau^e_{1,a}(\phi_{1,a}) + \tau^e_{1,b}(\phi_{1,b}) \\ \vdots \\ \tau^e_{n,a}(\phi_{n,a}) + \tau^e_{n,b}(\phi_{n,b}) \end{pmatrix} = \Pi\,\Gamma(\phi_1,\phi_2)\,,$$

with

$$\Pi = \text{diag}(\Pi_1,\ldots,\Pi_n) \in \mathbb{R}^{n\times 2np}\,,$$
$$\Gamma = (\gamma_1(\phi_{1,a},\phi_{1,b})^T,\ldots,\gamma_n(\phi_{n,a},\phi_{n,b})^T)^T \in \mathbb{R}^{2np},$$

and each

$$\Pi_i = (\alpha_{i,1},\ldots,\alpha_{i,p},\beta_{i,1},\ldots,\beta_{i,p}) \in \mathbb{R}^{1\times 2p}\,,$$
$$\gamma_i = (y_1(\phi_{i,a}),\ldots,y_p(\phi_{i,a}),y_1(\phi_{i,b}),\ldots,y_p(\phi_{i,b}))^T \in \mathbb{R}^{2p}.$$

Moreover, to attain a similar decomposition for the dynamic behavior of the joint stiffness matrix σ and, in parallel, avoid using link and motor speed data, it is algebraically convenient to adopt the time-integral S_i of the total i-th joint stiffness σ_i, i.e. $S_i = \int_0^t \sigma_i(\tau)d\tau$, as a state variable. Then, its time derivative is $\dot{S}_i = \sigma_i$, which, by virtue of (4), is the sum of two addends depending on the i-th agonistic and antagonistic deflections, respectively. Hence, such a derivative can be expressed via the same basis functions used above, i.e.

$$\dot{S}_i = \sigma_i = \sum_{k=1}^{p} (\mu_{i,k}\, y_k(\phi_{i,a}) + \nu_{i,k}\, y_k(\phi_{i,b})),$$

where $\mu_{i,k}$ and $\nu_{i,k}$ are suitable coefficients. Accordingly, one can write $\dot{S} = \Sigma\,\Gamma(\phi_1,\phi_2)$, with $\Sigma = \text{diag}(\Sigma_1,\ldots,\Sigma_n) \in \mathbb{R}^{n\times 2np}$ and each $\Sigma_i = (\mu_{i,1},\ldots,\mu_{i,p},\nu_{i,1},\ldots,\nu_{i,p}) \in \mathbb{R}^{1\times 2p}$.

Finally, putting all together, the dynamics of the *compound* configuration vector $(q^T,S^T)^T$ can be written in the following form with actuation factorization:

$$\begin{aligned} M(q)\,\ddot{q} + h(q,\dot{q}) &= \tau_{\text{ext}} - \Pi\,\Gamma(\phi_a,\phi_b)\,, \\ \dot{S} &= \Sigma\,\Gamma(\phi_a,\phi_b)\,, \end{aligned} \tag{9}$$

where the basis functions used to decompose the total elastic torque and its partial derivative operate as input functions.

B. Nominal and Uncertain Model Decomposition

The dynamics in (9) includes functions that may be uncertain or even unknown, i.e. the inertia matrix $M(q)$, the functions appearing in $h(q,\dot{q})$, the coefficients of Π and Σ in the actuation-related terms, and the external torque τ_{ext} are only partially known. Under this hypothesis, we seek for a convenient decomposition of (9) separating a minimal yet exactly known dynamics from the remainder uncertain one.

To this purpose, it can be assumed, without loss of generality, that $M(q)^{-1}$ can be expanded as the sum of an *invertible* known matrix $\bar{M}(q)^{-1}$ and a remainder uncertain one $\Delta M(q)^{-1}$. In addition, as the basis functions used to factorize the actuation-related terms can be freely chosen, they are available and, thus, the only uncertainty affects the values of the involved coefficients Π and Σ. Finally, no assumptions on the separability of known and unknown terms in $h(q,\dot{q})$ are made. In summary, it holds:

$$\begin{aligned} M(q)^{-1} &= \bar{M}(q)^{-1} + \Delta M(q)^{-1}\,, \\ \Pi = \bar{\Pi} + \Delta\Pi\,,&\quad \Sigma = \bar{\Sigma} + \Delta\Sigma, \end{aligned} \tag{10}$$

with $\bar{\Pi}$ and $\bar{\Sigma}$ being nominal known values and $\Delta\Pi$ and $\Delta\Sigma$ the uncertain ones. Having stated the previous assumptions, one can left-multiply the first equation of (9) by $M(q)^{-1}$ and, using (10), obtain

$$\begin{aligned} \ddot{q} &= M(q)^{-1}(\tau_{\text{ext}} - h(q,\dot{q})) - M(q)^{-1}\,\Pi\,\Gamma(\phi_a,\phi_b) \\ &= \left(\bar{M}(q)^{-1} + \Delta M(q)^{-1}\right)(\tau_{\text{ext}} - h(q,\dot{q}))\ + \\ &\quad - \left(\bar{M}(q)^{-1} + \Delta M(q)^{-1}\right)\Delta\Pi\,\Gamma(\phi_a,\phi_b)\ + \\ &\quad - \Delta M^{-1}(q)\,\bar{\Pi}\,\Gamma(\phi_a,\phi_b) - \bar{M}^{-1}(q)\,\bar{\Pi}\,\Gamma(\phi_a,\phi_b)\ , \\ \dot{S} &= \bar{\Sigma}\,\Gamma(\phi_a,\phi_b) + \Delta\Sigma\,\Gamma(\phi_a,\phi_b). \end{aligned}$$

As a next step, one can separate the terms that are fully known from the remaining ones that can be lumped together into a vector signal $w = (w_q^T, w_S^T)^T$, which will be considered as unknown disturbance. Specifically, defining

$$\begin{aligned} w_q &= M(q)^{-1}(\tau_{\text{ext}} - h(q,\dot{q}))\ + \\ &\quad - \left(M(q)^{-1}\Delta\Pi + \Delta M(q)^{-1}\,\bar{\Pi}\right)\Gamma(\phi_a,\phi_b)\,, \\ w_S &= \Delta\Sigma\,\Gamma(\phi_a,\phi_b), \end{aligned}$$

where matrix $M(q)^{-1}$ has been recombined where possible for compactness, leads to the sought nominal model with explicit unknown input disturbance:

$$\begin{aligned} \ddot{q} &= -\bar{M}(q)^{-1}\,\bar{\Pi}\,\Gamma(\phi_a,\phi_b) + w_q\,, \\ \dot{S} &= \bar{\Sigma}\,\Gamma(\phi_a,\phi_b) + w_S\,. \end{aligned} \tag{11}$$

Remark 1: As shown later, the inertia inverse $\bar{M}(q)^{-1}$ can be chosen quite arbitrarily, provided that it is invertible, thereby avoiding information loss on the right hand-side of (8). Accordingly, to make the synthesis of a linear input-state observer feasible, as done in the next section, $\bar{M}(q)^{-1}$ is further assumed to be constant, i.e. $\bar{M}(q)^{-1} = \bar{M}^{-1}$. The remainder part of the inertia inverse is then $\Delta M(q)^{-1} = M(q)^{-1} - \bar{M}^{-1}$.

IV. Input-State Observer Design for ASR

The second step of our strategy is to dynamically reconstruct the unknown signal w (or better its discrete-time version), acting in (11), so as to enable its subsequent compensation by a suitable robust controller later derived. Leveraging on the model reformulation described in the previous section, w can be estimated by a linear unknown input-state observer using a nominal state form of such model.

To this aim, given a sampling period T and a discrete time k, let the compound configuration vector be the system output, i.e. $Y = (q^T, S^T)^T \in \mathbb{R}^{2n}$, the output of the robust controller be the system (manipulable) input, i.e. $U = \Gamma(\phi_a, \phi_b) \in \mathbb{R}^{2np}$, and $w = (w_q^T, w_S^T)^T \in \mathbb{R}^{2n}$ be an unknown disturbance (note that no link speed data is used); defining the state vector $X = (q^T, \dot{q}^T, S^T)^T$, (11) is written as $\dot{X} = A_c X + B_c U + W_c w$, with $A_c = \begin{pmatrix} 0_n & \mathbb{I}_n & 0_n \\ 0_{2n\times n} & 0_{2n\times n} & 0_{2n\times n} \end{pmatrix}$, $B_c = (0_{np\times n}, -(\bar{M}^{-1}\bar{\Pi})^T, \bar{\Sigma}^T)^T$, $W_c = (0^T_{n\times 2n}, \mathbb{I}_{2n})^T$. Assuming U and w be constant between two consecutive discrete times and using the approach in [20] and the fact that A_c is nilpotent of order 2, i.e. $A_c^k = 0$ for $k \geq 2$, it is straightforward to obtain the linear discrete-time state form

$$X_{k+1} = A\,X_k + B\,U_k + W\,w_k\,, \quad Y_k = C\,X_k\,, \tag{12}$$

where $X_k = X(kT)$, $Y_k = Y(kT)$, $U_k = U(kT)$, and $w_k = w(kT)$ and where

$$A = e^{A_c T} = \mathbb{I}_{3n} + T A_c = \begin{pmatrix} \mathbb{I}_n & T\,\mathbb{I}_n & 0_n \\ 0_n & \mathbb{I}_n & 0_n \\ 0_n & 0_n & \mathbb{I}_n \end{pmatrix}, C = \begin{pmatrix} \mathbb{I}_n & 0_n & 0_n \\ 0_n & 0_n & \mathbb{I}_n \end{pmatrix}$$

$$B = J\,B_c = \begin{pmatrix} 0_{n\times np} \\ -T\bar{M}^{-1}\,\bar{\Pi} \\ T\bar{\Sigma} \end{pmatrix}, W = J\,W_c = \begin{pmatrix} 0_{n\times 2n} \\ T\,\mathbb{I}_{2n} \end{pmatrix}, \tag{13}$$

being $J = A_c^{-1}(e^{A_c T} - \mathbb{I}_{3n}) = T$.

Now, indicating with $\hat{X}_k = (\hat{q}(kT)^T, \dot{\hat{q}}(kT)^T, \hat{S}(kT)^T)^T$ an estimate of X_k, with $\hat{w}_k$ an estimate of w_k, with $\mathbb{Y}_k = (Y_{k-2}^T, Y_{k-1}^T, Y_k^T)^T$ the output history vector of the latest 3 values of Y_k, and with $\{\lambda_1, \ldots, \lambda_{3n}\}$ a set of constants that can be freely chosen within the unit circle of the complex plane, the linearity of (12) allows designing the following delayed estimator:

Theorem 1 (DUIO Design for ASR): Given the reformulated model of an articulated soft robot described in (12), with matrices A, B, and C as in (13), a 2-sample delayed state estimate $\hat{X}_{k-2}$ can be computed via the iterative rule

$$\hat{X}_{k-2+1} = E\,\hat{X}_{k-2} + B\,U_{k-2} + F\,(\mathbb{Y}_k + \mathbb{N}_k)\,, \tag{14}$$

in which

$$E = \mathrm{diag}(\lambda_1, \ldots, \lambda_{3n})\,,$$

$$F = \left(\begin{array}{c|c|c|c|c|c} F_1 - \mathbb{I}_n & \frac{1}{T}\mathbb{I}_n & 0_n & 0_n & 0_n & 0_n \\ \hline -F_1 - \frac{1}{T}\mathbb{I}_n & \mathbb{I}_n & \frac{1}{T}F_3 & 0_n & \frac{1}{T}\mathbb{I}_n & 0_n \\ \hline \frac{1}{T}\mathbb{I}_n & F_2 + \mathbb{I}_n & 0_n & \mathbb{I}_n & 0_n & 0_n \end{array}\right) \tag{15}$$

with $F_1 = \mathrm{diag}(\lambda_1, \ldots, \lambda_n)$, $F_2 = \mathrm{diag}(\lambda_{n+1}, \ldots, \lambda_{2n})$, $F_3 = \mathrm{diag}(\lambda_{2n+1}, \ldots, \lambda_{3n})$, and in which

$$\mathbb{N}_k = (0_{2n}, (N_1 U_{k-2})^T, (N_2 U_{k-2} + N_1 U_{k-1})^T)^T\,, \tag{16}$$

with $N_1 = \begin{pmatrix} 0_{n\times np} \\ -T\,\bar{\Sigma} \end{pmatrix}$ and $N_2 = \begin{pmatrix} T^2 \bar{M}^{-1}\,\bar{\Pi} \\ -T\,\bar{\Sigma} \end{pmatrix}$. Moreover, given $G = (0_{2n\times n}, \frac{1}{T}\,\mathbb{I}_{2n}, 0_{2n})$, a 2-sample delayed input estimate $\hat{w}_{k-2}$ can be computed via the formula

$$\hat{w}_{k-2} = G \begin{pmatrix} \hat{X}_{k-1} - A\,\hat{X}_{k-2} - B\,U_{k-2} \\ Y_k - C\hat{X}_{k-2} \end{pmatrix}. \tag{17}$$

Proof: The DUIO derivation can be achieved within the framework of delayed input-state observers (cf. e.g. [17]), which can be done through the following three steps.

1) Existence: The existence of the observer with a suitable delay L is ensured if, and only if, the compound state dynamics in (12) is strongly observable with respect to all initial conditions and invertible with respect to the unknown input w_k when the system output is Y_k [17]. Referring to the conditions in (6) and (7), the system dynamics has a null direct matrix, $D = 0_{2n}$, multiplying w_k, and hence the first three invertibility matrices in the sequence are

$$\mathbb{H}_0 = 0_{2n}, \mathbb{H}_1 = \left(\begin{array}{c|c} \multicolumn{1}{c|}{0_{2n}} & 0_{2n} \\ \hline \begin{matrix} 0_n & 0_n \\ 0_n & T\mathbb{I}_n \end{matrix} & 0_{2n} \end{array}\right),$$

$$\mathbb{H}_2 = \begin{pmatrix} 0_{2n} & 0_{2n} & 0_{2n} \\ CW & 0_{2n} & 0_{2n} \\ CAW & CW & 0_{2n} \end{pmatrix} = \left(\begin{array}{c|c|c} 0_{2n} & 0_{2n} & 0_{2n} \\ \hline \begin{matrix} 0_n & 0_n \\ 0_n & T\mathbb{I}_n \end{matrix} & 0_{2n} & 0_{2n} \\ \hline \begin{matrix} T^2\mathbb{I}_n & 0_n \\ 0_n & T\mathbb{I}_n \end{matrix} & \begin{matrix} 0_n & 0_n \\ 0_n & T\mathbb{I}_n \end{matrix} & 0_{2n} \end{array}\right).$$

Given that the dimension of the unknown input w_k is $m = 2n$ and that $\mathrm{rank}(\mathbb{H}_0) = 0$, $\mathrm{rank}(\mathbb{H}_1) = n$ and $\mathrm{rank}(\mathbb{H}_2) = 3n$, the conditions in (6) and (7) are first satisfied with $L = 2$, which is the (minimum) delay of the observer.

2a) State Estimation – Convergence Conditions: Knowing that the required delay is $L = 2$, one can define the output history vector $\mathbb{Y}_k = (Y_{k-2}^T, Y_{k-1}^T, Y_k^T)^T$ — comprising the latest $L + 1 = 3$ output samples and being the one in the theorem statement — and assume a state estimate update rule as in (14), where E and F are matrices to be suitably chosen and where $\mathbb{N}_k$ is still a free vector. As the observer is meant to provide, at any step k, an estimate of the past compound state X_{k-2}, the current state estimation error can be defined as $e_k = \hat{X}_{k-2} - X_{k-2}$. Shifting backward in time of 2 steps the robot dynamics in (12) yields $X_{k-2+1} = A\,X_{k-2} + B\,U_{k-2} + W\,w_{k-2}$, which allows, along with the assumed observer's update rule, deriving the following state estimation error dynamics:

$$\begin{aligned} e_{k+1} &= \hat{X}_{k-2+1} - X_{k-2+1} = \\ &= E\,\hat{X}_{k-2} - A\,X_{k-2} + F\,\mathbb{Y}'_k - W\,w_{k-2} = \\ &= E\,e_k + (E - A)\,X_{k-2} + F\,\mathbb{Y}'_k - W\,w_{k-2}\,, \end{aligned} \tag{18}$$

where the addend $E\,X_{k-2}$ has been added and subtracted in the last line of (18), and where $\mathbb{Y}'_k = \mathbb{Y}_k + \mathbb{N}_k$.

Now, the observer's convergence must be guaranteed for all delayed states X_{k-2} and all behaviors of the delayed unknown input w_{k-2}, and hence (18) should be made independent of them. A possible dynamics that has the desired convergence property and that is algebraically compliant with (18) is $e_{k+1} = E\,e_k$ with E a Schur matrix. By comparing the two expressions one gets the condition

$$(E - A)\,X_{k-2} + F\,\mathbb{Y}'_k - W\,w_{k-2} = 0\,. \tag{19}$$

To remove the explicit dependency of this expression on the output history vector $\mathbb{Y}_k$, its entries can be expanded in terms of the delayed state X_{k-2} and the latest unknown input samples w_{k-2}, w_{k-1}, and w_k. This leads to the three formulas:

$$\begin{aligned} Y_{k-2} &= C\,X_{k-2}\,, \\ Y_{k-1} &= CA\,X_{k-2} + CB\,U_{k-2} + CW\,w_{k-2}\,, \\ Y_k &= CA^2\,X_{k-2} + CAB\,U_{k-2} + CB\,U_{k-1} \\ &\quad + CAW\,w_{k-2} + CW\,w_{k-1}\,, \end{aligned}$$

from which, after conveniently choosing matrix $\mathbb{N}_k$ as

$$\mathbb{N}_k = -\begin{pmatrix} 0_{2n} & 0_{2n} & 0_{2n} \\ CB & 0_{2n} & 0_{2n} \\ CAB & CB & 0_{2n} \end{pmatrix} \begin{pmatrix} U_{k-2} \\ U_{k-1} \\ U_k \end{pmatrix},$$

whose direct computation leads to its expression in (16), with $N_1 = -CB$ and $N_2 = -CAB$, one can write

$$\begin{pmatrix} Y'_{k-2} \\ Y'_{k-1} \\ Y'_k \end{pmatrix} = \begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix} X_{k-2} + \begin{pmatrix} 0_{2n} & 0_{2n} & 0_{2n} \\ CW & 0_{2n} & 0_{2n} \\ CAW & CW & 0_{2n} \end{pmatrix} \begin{pmatrix} w_{k-2} \\ w_{k-1} \\ w_k \end{pmatrix}$$

or, in matrix form, $\mathbb{Y}'_k = \mathcal{O}_2 X_{k-2} + \mathbb{H}_2 \mathbb{W}_k$, where $\mathcal{O}_2$ is the 2-step observability matrix and $\mathbb{W}_k = (w_{k-2}^T, w_{k-1}^T, w_k^T)^T$. Consequently, (19) becomes

$$(E - A + F\mathcal{O}_2) X_{k-2} + (F\,\mathbb{H}_2 - (W, 0_{2n}, 0_{2n}))\,\mathbb{W}_k = 0\,.$$

Now, in order to satisfy this expression for every X_{k-2} and $\mathbb{W}_k$, it must be that

$$E = A - F\mathcal{O}_2 \quad \text{and} \quad F\,\mathbb{H}_2 = (W, 0_{2n}, 0_{2n})\,. \tag{20}$$

2b) State Estimation – Derivation of the Matrices: The second condition in (20) requires that F is in the left-nullspace of the last m columns of $\mathbb{H}_2$, that are $P = (0_{4n}, \mathbb{H}_1^T)^T$. Given a matrix $\bar{N}$ whose rows are a basis of the left-nullspace of $\mathbb{H}_1$, the rows of matrix $\text{diag}(\mathbb{I}_{2n}, \bar{N})$ are a basis of the left-nullspace of P. Seeing the structure of $\mathbb{H}_1$, it suffices to choose $\bar{N} = (\mathbb{I}_{3n}, 0_{3n\times n})$. Moreover, given an invertible matrix W^*, we can define $N = W^*\text{diag}(\mathbb{I}_{2n}, \bar{N})$, whose rows also form a basis of the left nullspace of P. Therefore, to find W, first note that $N\begin{pmatrix} 0_{2n} & 0_{2n\times 4n} \\ \mathcal{O}_1 W & \mathbb{H}_1 \end{pmatrix} = W^*\begin{pmatrix} 0_{2n} & 0_{2n} \\ \bar{N}\,\mathcal{O}_1 W & 0_{2n} \end{pmatrix} = W^*\,V$, where $\mathcal{O}_1 = (C^T, (CA)^T)^T$. As the required delay is $L = 2$, the first $2n$ columns of $\mathbb{H}_2$ are linearly independent and hence $\text{rank}(V) = 2n$. Matrix W^* can be chosen so that its last $2n$ rows are a left-inverse of V, while the top ones are a basis of its left-nullspace. The choice

$$W^* = \left(\begin{array}{c|c|c} \multicolumn{2}{c}{\mathbb{I}_{3n}} & 0_{3n\times 2n} \\ \hline 0_{2n\times 3n} & \begin{array}{c} 0_n \\ \frac{1}{T}\,\mathbb{I}_n \end{array} & \begin{array}{c} \mathbb{I}_n/T^2 \\ 0_n \end{array} \end{array}\right),$$

leads to a matrix N satisfying the expression $N\,\mathbb{H}_2 = \begin{pmatrix} 0_{2n} & 0_{2n} \\ \mathbb{I}_{2n} & 0_{2n} \end{pmatrix}$ and then $N = W^*$. Based again on the structure of (20) and the columns of W, F can be factorized as $F = F^*N$, with $F^* = (F_1^*, F_2^*)$, so that (20) itself can be written as $(F_1^*, F_2^*)\begin{pmatrix} 0_{2n} & 0_{2n} \\ \mathbb{I}_{2n} & 0_{2n} \end{pmatrix} = (W, 0_{3n\times 2n})$, from which it follows $F_2^* = W$, while F_1^* is still free. Plugging F into the first condition in (20) yields $E = A - (F_1^*, W)\,N\mathcal{O}_2 = A - (F_1^*, W)(R^T, Q^T)^T$, with

$$R = \begin{pmatrix} \mathbb{I}_n & 0_n & 0_n \\ 0_n & 0_n & \mathbb{I}_n \\ \mathbb{I}_n & T\mathbb{I}_n & 0_n \end{pmatrix},\quad Q = \begin{pmatrix} \frac{1}{T^2}\mathbb{I}_n & \frac{2}{T}\mathbb{I}_n & 0_n \\ 0_n & 0_n & \frac{1}{T}\mathbb{I}_n \end{pmatrix},$$

and then

$$E = A - WQ - F_1^* R =$$
$$= \begin{pmatrix} 0_n & \mathbb{I}_n & 0_n \\ -\frac{1}{T^2}\mathbb{I}_n & -\frac{2}{T}\mathbb{I}_n & 0_n \\ 0_n & 0_n & -\frac{1}{T}\mathbb{I}_n \end{pmatrix} - F_1^* \begin{pmatrix} \mathbb{I}_n & 0_n & 0_n \\ 0_n & 0_n & \mathbb{I}_n \\ \mathbb{I}_n & T\mathbb{I}_n & 0_n \end{pmatrix}.$$

To finally ensure that E is Schur and diagonal as in (15), it suffices to choose $F_1^* = (A - WQ - \text{diag}(\lambda_1, \ldots, \lambda_{3n}))R$, which also leads to F's expression in (15).

3) Unknown Input Reconstruction: Finally, an estimate, that is optimal in the least square sense, of the delayed unknown input w_{k-2} can be retrieved from the conditions:

$$\hat{X}_{k-2+1} - X_{k-2+1} = \hat{X}_{k-2+1} - AX_{k-2} - B\,U_{k-2} - Ww_{k-2}\,,$$
$$\hat{Y}_{k-2} - Y_{k-2} = C\hat{X}_{k-2} - Y_{k-2}\,.$$

Once the delayed state estimate $\hat{X}_{k-2}$ has converged to X_{k-2}, the right hand-sides of these equations converge to zero; then the remaining expressions can be rearranges as follows:

$$\begin{pmatrix} W \\ 0_{2n} \end{pmatrix} w_{k-2} = \begin{pmatrix} \hat{X}_{k-2+1} - AX_{k-2} - B\,U_{k-2} \\ Y_{k-2} - C\hat{X}_{k-2} \end{pmatrix}.$$

Left-multiplying both sides of this equation by a matrix $G = (W^TW)^{-1}(W^T, 0_{2n})$, whose computation leads to the formula in theorem's statement, allows obtaining the sought (17), which concludes the proof. □

V. Robust Control and Closed-Loop Stability

Given desired trajectories, $q_d(t)$ and $\sigma_d(t)$, for the compound configuration, define the desired state signal samples $X_{d,k} = (q_d(kT)^T, \dot{q}_d(kT)^T, S_d(kT)^T)^T$, with $S_d(t) = \int_0^t \sigma_d(\tau)d\tau$. Then, using the information retrieved by the DUIO, the following can be found:

Theorem 2: Given an articulated soft robot as in (8), the feedback-feedforward control law

$$U_k = K(\hat{X}_{k-2} - X_{d,k}) + P(X_{d,k+1} - A\,X_{d,k} - W\,\hat{w}_{k-2})\,, \tag{21}$$

where $\hat{X}_{k-2}$ and $\hat{w}_{k-2}$ are found using (14) and (17), $P = (B^TB)^{-1}B^T$, and K is such that $A + BK$ is Schur, ensures global and robust bounded stability for the tracking error of desired position and stiffness trajectories, $q_d(t)$ and $\sigma_d(t)$.

Proof: The dynamics of the discrete-time tracking error, $Z_k = X_k - X_{d,k}$, reads

$$\begin{aligned} Z_{k+1} &= AX_k + BU_k + Ww_k - X_{d,k+1} = \\ &= AZ_k + BU_k - X_{d,k+1} + AX_{d,k} + Ww_k\,. \end{aligned} \tag{22}$$

A desired convergent dynamics for the state vector is $Z_{k+1} = A^*Z_k$, with $A^* = A + B\,K$, where $K \in \mathbb{R}^{2np\times 3n}$ is a free gain matrix that can be chosen to make matrix A^* Schur. Comparing such desired expression with (22) yields the condition $BU_k = BKZ_k + X_{d,k+1} - AX_{d,k} - Ww_k$. In practice, as only the estimates of X_k and w_k are available with 2-sample delays, the following best-effort condition can be ensured:

$$BU_k = BK\hat{Z}_k + X_{d,k+1} - AX_{d,k} - W\hat{w}_{k-2}\,, \tag{23}$$

with $\hat{Z}_k = \hat{X}_{k-2} - X_{d,k}$. The overall closed-loop dynamics, including the tracking error dynamics in (22) and the estimation error e_k from Theorem 1 reads

$$\begin{pmatrix} Z_{k+1} \\ e_{k+1} \end{pmatrix} = \begin{pmatrix} A^* & 0_{3n} \\ 0_{3n} & E \end{pmatrix} \begin{pmatrix} Z_k \\ e_k \end{pmatrix} + \begin{pmatrix} \varphi_k \\ 0_{3n} \end{pmatrix}, \tag{24}$$

with $\varphi_k = BK(\hat{Z}_k - Z_k) + W\tilde{w}_k$ and $\tilde{w}_k = w_k - \hat{w}_{k-2}$. It can be seen, as expected by its design, that the dynamics of the estimation error e_k is independent of U_k and w_k, and hence its closed-loop convergence is ensured by E being Schur. That being so, after a transient in the observer estimates, $\hat{X}_{k-2} \simeq X_{k-2}$ and $\hat{w}_{k-2} \simeq w_{k-2}$. Moreover, considering the small delay $L = 2$ [21] and assuming a small enough sampling period T, it also holds $\hat{X}_{k-2} \simeq X_k$, $U_{k-2} \simeq U_k$, $w_{k-2} \simeq w_k$, $Y_{k-2} \simeq Y_k$. As a result, $\hat{Z}_k \simeq Z_k$ and $\tilde{w}_k \simeq w_k - \hat{w}_k \simeq 0$ and the forcing

TABLE I
NOMINAL PARAMETERS OF THE VSA-DRIVEN ROBOT

Link masses	$m_1 = m_2 = m_3 = 0.45$ kg
Link inertia	$I_1 = I_2 = 0.0045$ kgm^2, $I_3 = 0.001125$ kgm^2
Link lengths	$l_1 = l_2 = 0.1$ m, $l_3 = 0.05$ m
Agonistic Antagonistic	$k_a = 0.0026$ Nm, $a_a = 8.9995$ rad^{-1} $k_b = 0.0011$ Nm, $a_b = 8.9989$ rad^{-1}

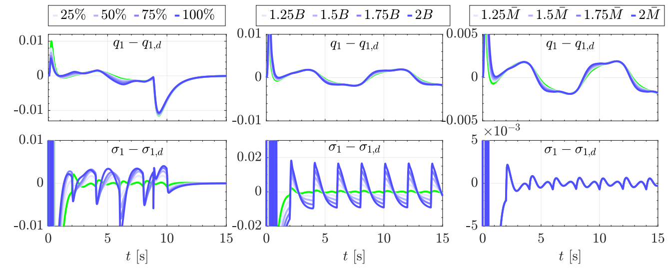

Fig. 2. *Robustness to parametric (left), input matrix B (middle) and inertia matrix $\bar{M}$ (right) uncertainties, ranging from 25% to 100% deviation from their nominal values*. Green indicates the closed-loop behavior with nominal choices for parameters, B and $\bar{M}$. Position and stiffness tracking errors grow as uncertainty increases, yet, the closed-loop system performance does not deteriorate considerably. Tracking errors exist during time-varying references and are due to worse feedforward compensation, originating from larger parametric uncertainty and model simplification. Indeed, they nicely converge when references become still (last phase of left plots).

term φ_k in the closed-loop dynamics vanishes, i.e. φ_k tends to zero. More precisely, it can be shown that $\hat{Z}_{k-1} - Z_{k-1} = A(\hat{Z}_{k-2} - Z_{k-2}) + W(\hat{w}_{k-2} - w_{k-2}) \simeq 0$, which in turn implies $\hat{Z}_k - Z_k = A(\hat{Z}_{k-1} - Z_{k-1}) + W(\hat{w}_{k-1} - w_{k-1}) \simeq W(\hat{w}_{k-1} - w_{k-1})$. As (A, B) is controllable, a matrix K making A^* Schur always exists, thus ensuring $||A^* Z_k|| < ||Z_k||$. Also, for small T [21], $\hat{w}_{k-1} \simeq \hat{w}_{k-2} \simeq w_{k-2}$. Then, from (24), Z_k evolves with a stable dynamics subject to a forcing signal φ such that $||\varphi|| = ||BKW(w_{k-2} - w_{k-1}) + W(w_k - w_{k-2})|| \leq T(||B||\,||K|| + 2)\gamma$, where γ is the absolute maximum variation of w_k between any two consecutive samples. A possible estimate for this quantity is $\gamma = \max(\ddot{q}_{max}, \dot{S}_{max}) = \max(\ddot{q}_{max}, \sigma_{max})$, where $\ddot{q}_{max}$ and σ_{max} are the maximum reachable acceleration and stiffness. Z_k converges to the equilibrium $Z_k = (\mathbb{I}_{3n} - A^*)^{-1}\varphi$, whose norm is $||Z_k|| \leq T\,||(\mathbb{I}_{3n} - A^*)^{-1}||(||B||\,||K|| + 2)\gamma$ and hence is upper bounded by a quantity proportional to T. Finally, left-multiplying both members of (23) by the pseudoinverse of B, the control law in (21) is obtained. □

VI. METHOD APPLICATION AND EVALUATION

This section shows the performance of the proposed solution when applied to the 3-degree-of-freedom ASR in Fig. 1. In the considered hardware setup, each joint of the robot is driven by a *qbmove advanced* actuator [22] that allows simultaneous settings of link position and joint stiffness. The section is organized in three parts: 1) instances of the proposed estimator and robust controller are derived based on a coarse but very convenient simplification of the robot's compound dynamics; 2) their effectiveness and robustness are shown in simulation with large model uncertainty due to the few amount of information used for their derivation; 3) experimental test results with the adopted real hardware setup are reported.

A. Derivation of Estimator and Controller Components

To test the robustness of the method and to show also the few amount of information used to derive the corresponding estimator and controller, first of all, all links' interactions are completely neglected, thereby leading to a nominal inertia matrix inverse that is diagonal and given by $\bar{M}(q)^{-1} = \mathrm{diag}(1/I_1, 1/I_2, 1/I_3)$ (cf. e.g. [23]), where I_i are the solely link inertia constants whose values are reported in Table I. Secondly, from [22], the i-th VSA device can apply the total elastic torque and set the joint stiffness given by the following VSA-specific formulas

$$\begin{aligned} \tau_i^e &= k_{i,a}\sinh(u_{i,a}) + k_{i,b}\sinh(u_{i,b}), \\ \sigma_i &= a_{i,a}k_{i,a}\cosh(u_{i,a}) + a_{i,b}k_{i,b}\cosh(u_{i,b}), \end{aligned} \tag{25}$$

where $u_{i,a} = a_{i,a}\phi_{i,a}$, $u_{i,b} = a_{i,b}\phi_{i,b}$, and where $k_{i,a}$, $k_{i,b}$, $a_{i,a}$, and $a_{i,b}$ are suitable constants whose nominal values k_a, k_b, a_a, and a_b, experimentally identified by the manufacturer, are reported again in Table I. Given the convex behavior of such functions, one can expand them as

$$\tau_i^e = \pi_i(t)(u_{i,a} + u_{i,b}), \ \sigma_i = \mu_i(t)(u_{i,a} + u_{i,b}), \tag{26}$$

where $\pi_i(t)$ and $\mu_i(t)$ are time-varying yet bounded slope signals. While for our approach any positive constant value can be used in place of these signals, it is reasonable to conservatively tune them bargaining the maximum elastic torque $\tau^e_{\max}$ and stiffness $\sigma_{\max}$ demands. To do so, using the prosthaphaeresis formulas for hyperbolic functions, with $a_{i,a} = a_{i,b} = a$ and $k_{i,a} = k_{i,b} = k$, one can rewrite (25) as $\tau_i^e = 2k\,\kappa\,\omega$ and $\sigma_i = 2ak\,\varepsilon\,\omega$, with $\kappa = \sinh((u_{i,a} + u_{i,b})/2)$, $\omega = \cosh((u_{i,a} - u_{i,b})/2)$, and $\varepsilon = \cosh((u_{i,a} + u_{i,b})/2)$. The ratio $\varepsilon/\kappa = \coth((u_{i,a} + u_{i,b})/2) = \sigma_i/(a\tau_i^e)$ allows finding $(u_{i,a} + u_{i,b})/2 = \mathrm{arccoth}(\sigma_i/(a\tau_i^e)) = \chi_1$. Then, from the relation $\omega = \tau_i^e/(2k\,\kappa)$, one also obtains $(u_{i,a} - u_{i,b})/2 = \mathrm{arccosh}(\tau_i^e/(2k\sinh(\chi_1))) = \chi_2$, and finally $\begin{pmatrix} u_{i,a} \\ u_{i,b} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}^{-1} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$. Evaluating this last relation for the maximum torque and stiffness demands allows obtaining the maximum values for $u_{i,a}$ and $u_{i,b}$, which are then converted to the maximum of π_i and μ_i. For the adopted qbmove actuators, it holds $\tau^e_{\max} = 7$ Nm and $\sigma_{\max} = 83.5$ Nm/rad, and hence it holds $\pi_i = 3.3416$ and $\mu_i = 37.2032$.

In accordance with this reasoning and the proposed formalization, the nominal matrices of the actuation-related terms are chosen as $\bar{\Pi} = \mathrm{diag}(\bar{\Pi}_1, \bar{\Pi}_2, \bar{\Pi}_3)$, $\bar{\Sigma} = \mathrm{diag}(\bar{\Sigma}_1, \bar{\Sigma}_2, \bar{\Sigma}_3)$, with $\bar{\Pi}_i = (\pi_i, \pi_i, 0, 0)$ and $\bar{\Sigma}_i = (0, 0, \mu_i, \mu_i)$, for all i, and the (manipulable) input vector is

$$\Gamma(\phi_a, \phi_b) = (\gamma_1^T(\phi_{1,a}, \phi_{1,b}), \gamma_2^T(\phi_{2,a}, \phi_{2,b}), \gamma_3^T(\phi_{3,a}, \phi_{3,b}))^T,$$

with $\gamma_i = (u_{i,a}, u_{i,b})^T$, for all i. Accordingly, the (manipulable) input vector is $U_k = \Gamma(\phi_a(kT), \phi_b(kT))$ and the matrices of the estimator are of immediate writing from Theorem 1, while they are omitted here for the sake of space. Finally, it should be recalled, as stated in Section III, that the actual inputs to the i-th VSA device are the motor positions, $\theta_{i,a}$ and $\theta_{i,b}$. They are finally determined by inverting (1) and are given by $\theta^c_{i,a} = q_i - u_{i,a}/a$ and $\theta^c_{i,b} = q_i - u_{i,b}/a$.

B. Effectiveness and Robustness Evaluation

To show its validity and robustness, the proposed approach is tested in Matlab/Simulink with parametric, nominal inertia matrix $\bar{M}$, and input matrix B uncertainties. Nominal parameters are reported in Table I. Nominal matrix $\bar{M}$ is set to be $\mathrm{diag}(I_1, I_2, I_3)$ representing the case where all off-diagonal

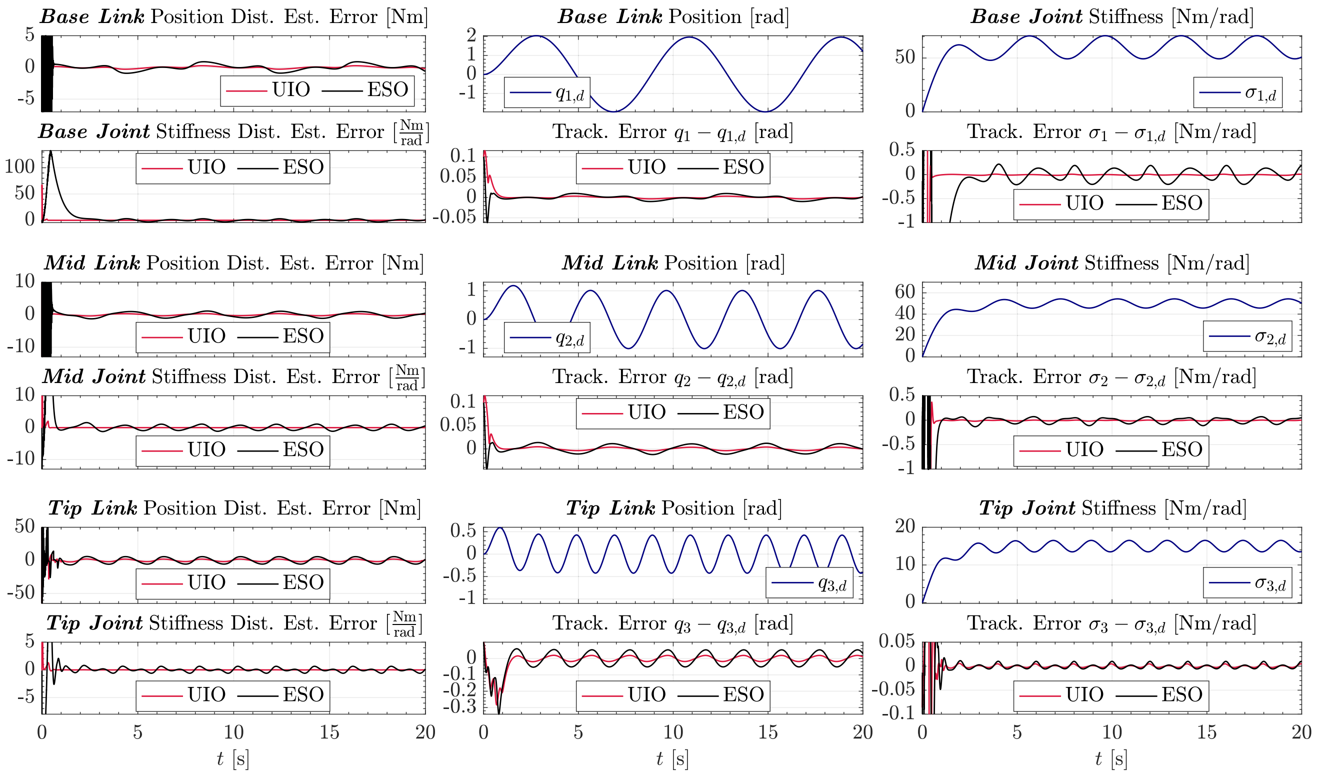

Fig. 3. *Robustness to large uncertainty and comparison with ESO*. Results from typical simulation runs with 100% deviation of system parameters from the nominal values, as well as a drastic simplification of the system model. Despite the large uncertainty, when the proposed DUIO-solution is used, the desired trajectories are accurately tracked as a result of fast and precise estimation of the disturbance perturbing the nominal system. The ESO-based solution under-performs especially when the disturbance signals are rapidly changing.

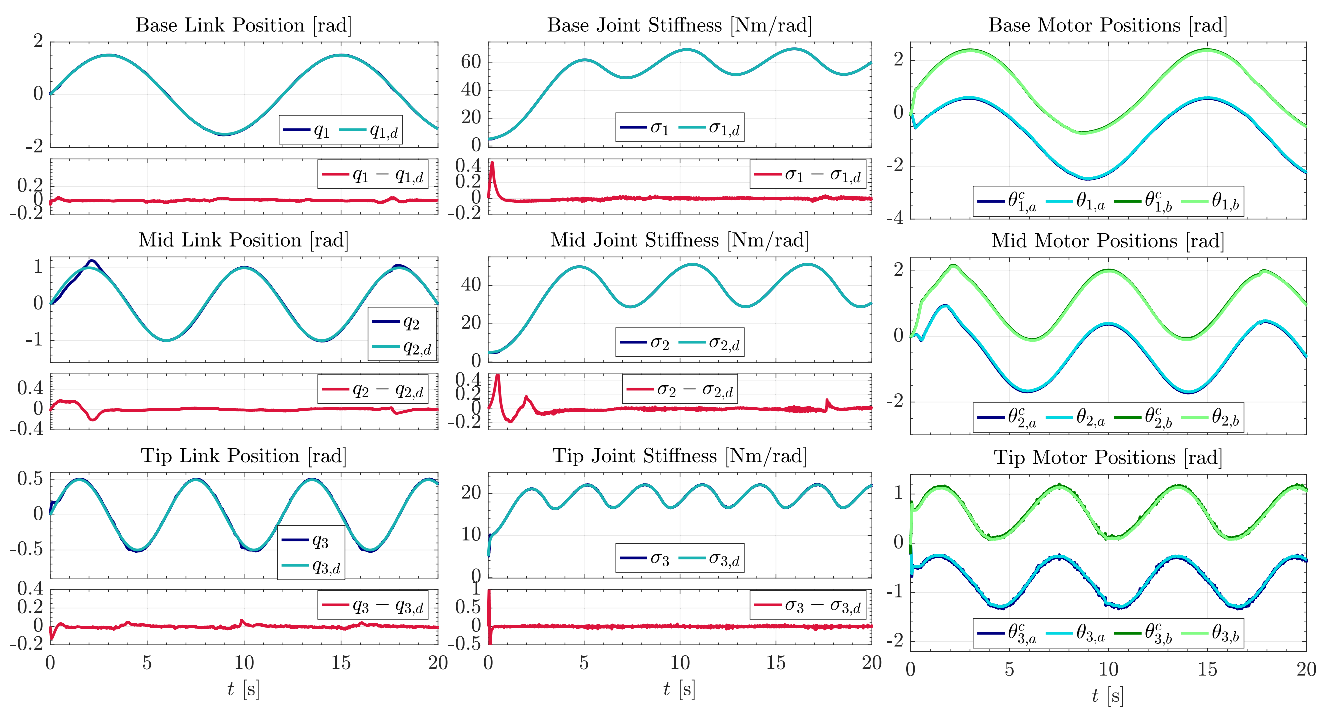

Fig. 4. *Experimental validation*. Results from testing with real ASR pictured in Fig. 1, when the task is to simultaneously track sinusoidal position and stiffness trajectories. The position and stiffness tracking errors are reported in red for each degree-of-freedom. Despite the drastically simplified robot model, upon which the estimator from Theorem 1 and the controller from Theorem 2 are derived, the proposed solution can robustly track the desired signals. Commanded $\theta^c_{i,j}$ and executed $\theta_{i,j}$ motor angles are shown and largely within the available range of the involved VSA devices.

entries of $M(q)$ are neglected. Perturbation of parameters, $\bar{M}$, and B range from a 25% to a 100% increase from their nominal values. Fig. 2 depicts the position and stiffness tracking errors for the robot's base segment, being the one with the most visible trends. A common behavior is that higher tracking errors appear with larger uncertainty, yet, the closed-loop performance is not significantly deteriorated even with substantial uncertainty. Noticeably, this shows that the method is robust with respect to the choice of $\bar{M}$ and B, which, in practice, allows neglecting all off-diagonal entries of $M(q)$ and approximating the diagonal ones even up to a relative error of 100%.

Moreover, the proposed solution is compared in Fig. 3 with an ESO-based one [24], initialized with position and stiffness tracking errors of (0.1,0.1,0.1) rad and (4,4,4) Nm/rad. The robust DUIO-based control law in (21) is used with the same control gains and, specifically, matrix K is chosen so that the closed loop matrix A^* are in the unit circle, and with desired references of the form $q_{i,d} = Q_i \sin \omega_{q_i} t$ and $\sigma_{i,d} = \epsilon_i + A_{\sigma_i} |\sin \omega_{\sigma_i} t|$, for $i \in \{1, 2, 3\}$. Fig. 3 shows that the UIO solution outperforms the ESO one, achieves a faster and more precise disturbance estimation and allows better tracking errors.

C. Experimental Validation

An experimental validation of the proposed solution is presented using the real hardware pictured in Fig. 1. Nominal geometric and inertial parameter values are in Table I and the desired trajectories have the same form as in the simulations, so as to capture similar behaviors, and have been chosen with various amplitudes and frequencies for each joint. The controller is also set as in Section VI-A. Link positions are measured through the encoders embedded in the *qbmove* actuators, while joint stiffness is obtained through the use of the stiffness estimator in [25]. As it can be seen in Fig. 4, the proposed solution robustly tracks the desired signals, and imperfect yet practically negligible tracking errors occur only very rarely, which is due to intrinsic slackness of the actuators. Noteworthy, despite the drastically simplified robot model, used to design the estimator from Theorem 1 and the controller from Theorem 2, each link position and joint stiffness are smoothly and successfully controlled.

VII. Conclusion

This letter formulated the model of VSA-driven articulated soft robots with the factorized actuator matrix suitable for the decomposition into the nominal and uncertain dynamics, and it derived a novel solution for their control. Using delayed unknown input observer theory and disturbance-observer-based control, the proposed approach enables successful position and stiffness tracking, even with large disturbance due to the model uncertainty. Validation of the method was carried out in simulations and experiments, proving also its robustness to parametric and structural uncertainties. Future work will explore the advantages of information decoupling between possible external interaction and uncertainty estimation, due to parametric changes and actuator failures.

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